

# SOLVING LINEAR SYSTEMS USING SUBSTITUTION

 Guide Notes

**LINEAR SYSTEM OF EQUATIONS:** is a set of equations with the same pair of variables. When we are solving systems using the **Substitution Method**, we have to choose one of the equations and solve for one variable (the easiest one) and then plug it into the other equation by substituting the chosen variable and solving for the other.

For two variable systems, there are three possible types: Independent, inconsistent and dependent.

1. 
$$\begin{cases} x = 3y - 1 \\ 3x - y = 2 \end{cases}$$

4. 
$$\begin{cases} 4x - 3y = 18 \\ y + 2 = 0 \end{cases}$$

2. 
$$\begin{cases} -3x + 3y = 4 \\ -x + y = 3 \end{cases}$$

5. 
$$\begin{cases} x + y = 4 \\ 5x - 4y = 6 \end{cases}$$

3. 
$$\begin{cases} 6x + 2y = 7 \\ y = 1 - 3x \end{cases}$$

6. 
$$\begin{cases} 6x - y = 3 \\ 5x - 2y = -8 \end{cases}$$

**INDEPENDENT SYSTEM** is a system where two distinct non-parallel lines intersect at one specific point (x,y).

Systems:

1	4	5	6
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**INCONSISTENT SYSTEM** is a system where two distinct lines are parallel. Since parallel lines never intersect, then there can be no solution.

Systems:

2	3
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**DEPENDENT SYSTEM** is a system which has infinite solutions.

System:

There is no system

**Sample Problems:** Find the solution of the following system by substitution

$$\begin{array}{lcl} 1. & \left\{ \begin{array}{l} x + y = 7 \\ 3x - y = -3 \end{array} \right. & \begin{array}{l} \text{(I)} \\ \text{(II)} \end{array} \end{array}$$

We choose the equation which contains the easiest variable to solve. In this case we select to solve variable “y” from equation I and then substitute it in equation II to find the value of the other variable, like follows:

$$y = 7 - x$$

Substituting in II:

$$3x - (7 - x) = -3$$

$$\text{Applying distributive property: } 3x - 7 + x = -3 \quad \rightarrow \quad 4x = 4 \quad \rightarrow \quad x = 1$$

Now, we calculate the value of variable “y” by substituting the result of “x” into the equation  $y = 7 - x$

$$y = 7 - 1 = 6$$

**Solution (1, 6). Independent System**

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$$\begin{cases} 2. & \begin{cases} 4x - y = 8 & \text{(I)} \\ x + y = 12 & \text{(II)} \end{cases} \end{cases}$$

We choose the equation which contains the easiest variable to solve. In this case we select to solve variable “y” from equation II and then substitute it in equation I to find the value of the other variable, like follows:

$$y = 12 - x$$

Substituting in I:

$$4x - (12 - x) = 8$$

Applying distributive property:  $4x - 12 + x = 8 \rightarrow 5x = 20 \rightarrow x = 4$

Now, we calculate the value of variable “y” by substituting the result of “x” into the equation  $y = 12 - x$

$$y = 12 - 4 = 8$$

**Solution (4, 8). Independent System**

$$\begin{cases} 3. & \begin{cases} -2x + 2y = 5 & \text{(I)} \\ -x + y = 4 & \text{(II)} \end{cases} \end{cases}$$

We choose the equation which contains the easiest variable to solve. In this case we select to solve variable “y” from equation II and then substitute it in equation I to find the value of the other variable, like follows:

$$y = 4 + x$$

Substituting in I:

$$-2x + 2(4 + x) = 5$$

Applying distributive property:  $-2x + 8 + 2x = 5 \rightarrow 0 = -3$

**No Solution. Inconsistent System**